‘Binary Pulse Amplitude Modulation Communication System’

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Abstract - In this case study, the basics of communications systems are explored and anaylzed. Several messages were sent across the coommunication channels across different frequencies and the results were recorded and analytically dissected. Binary messages were successfully modulated using pulse shapes and several pulse shapes were experimited with and compared. The error rate of the original and received messages was compared and the amount of bandwidth used was noted. Furthermore, the topic of sampling was discussed and explored as well as explaing the Nyquist Theorem through experimentation and modeling. Through this case study, the model for the PAM system was made with MATLAB and made open for any future changes or improvements. Furthermore, the modeling was efficient and all the parameters can easily be changed making modeling much more efficient and automated. The case study was concluded with all messages being sent and received through the channel with no errors.

I. Background

Communication Systems are everywhere in the modern world and are often specialized through protocols and standards to the type and amount of information that they process. In this study a Binary Pulse Amplitude Modulation (PAM) System will be modeled and tested with different parameters such as pulse shape, sampling speed, and allowed bandwidth. PAM systems are often used to model binary signals because the rate of the bits can be controlled and there are only two states making it much easier to discern what value the bit holds. Because binary is a relatively easy signal to process, it allows this study to make conclusions about the other parameters that the PAM system depends on such as the sampling rate, the bit rate, and the pulse shape. These parameters will be changed and explored with the goal of finding and proving known mathematical relationships as well as experimentally showing how changes in each parameter affect the entire system. The final challenge of this study will be to send a message in English through the system and process the received signal to recreate the original message. This will be a good test that uses all the different steps of the PAM system as well as serving to compare the performances of different parameters.

II. Methods

The goal for this study is to analyze how the triangular pulse and sinc pulse differ when used in a PAM system.

A. *Pulse Amplitude Modulation (PAM)*

A binary message made up of 1’s and 0’s is converted into a message made up of 1’s and -1’s. This creates a larger difference between the two values making it easier to distinguish if the bit is “on” or “off”.

The 1’s and -1’s are then modulated using a pulse shape. This is a big design choice as the pulse shape has a big impact on the amount of bandwidth that the communication channel will need. Equation 1 represents this mathematically using x_n binary message, p(t) pulse shape and T_s the symbol period.

$$\boldsymbol{y(t) = \sum_{n=-\infty}^{\infty} x_n p(t - nT_s)} \qquad \textbf{(1)}$$

B. *Nyquist Sampling Theorem*

The Nyquist Sampling Theorem states that you must sample the message at least twice as fast as the highest frequency component present in the message in order to be able to reconstruct the original signal from the sampled message. Equation 2 mathematically expresses this important property with f_s, the sampling frequency and f_m, the highest frequency present in original signal.

$$\boldsymbol{f_s \geq 2f_m} \qquad \textbf{(2)}$$

C. *Key variables*

These are the parameters that will be changed strategically to analyze their effect on the system. This analysis can be found at the end of the Results section.

T_p = Half of Triangular pulse width = 0.1 seconds

B_r = Bit Rate = $1/(2T_p)$ = 5 Bits/second

Bit Period = $1/B_r$ = 0.2 seconds

F_s = Sampling Frequency = $50/T_p$ = 500 samples/second

Sampling Period = $1/F_s$ = 0.002 seconds

III. Results

The basic building blocks of a communication system will quickly be explained with emphasis to the parts showcased in this study, a diagram of the major components can be observed in Figure 1. Figures from the MATLAB Demonstration will be used to show each step of the process from the sent message to the original message.

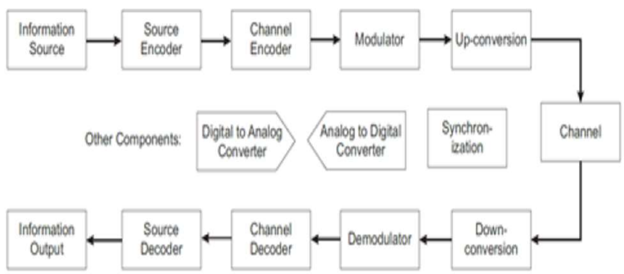

Figure 1: Block Diagram of Single User Digital Communication System

A. The Pulse Shapes P(t)

Two Pulse shapes were used in this case study to modulate the binary message.

a) Triangular pulse

The triangular pulse shown in Figure 2 was chosen as the first pulse shape modeled in this study.

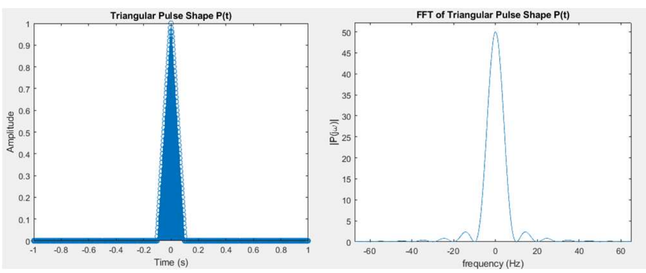

Figure 2: Triangular Pulse P(t) in time and frequency

Although the pulse shape is thin in the time domain, it is very wide in the frequency domain, which negatively impacts bandwidth as shown in Figure 2.

b) Sinc Pulse

The sinc pulse was set up to have a zero crossing at the same points as the binary impulse, thus causing the binary values to stand out.

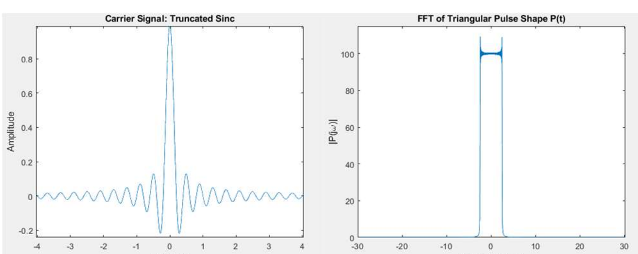

Figure 3: Sinc Pulse in and in frequency

It can be seen from figure 3, while the sinc pulse is very wide in time it is very narrow in frequency using less bandwidth.

B. Creating the Binary Message

The binary message in Figure 4 was created using a random number generator that gave either a 1 or 0. The Bit Period was the length of the triangular pulse which was set to 0.2 seconds and the bit rate was 5 bits/sec. The sampling frequency was 500 samples/sec. This was the recommended sampling frequency and exceeds the Nyquist Theorem requirements and helped provide clarity in the graph.

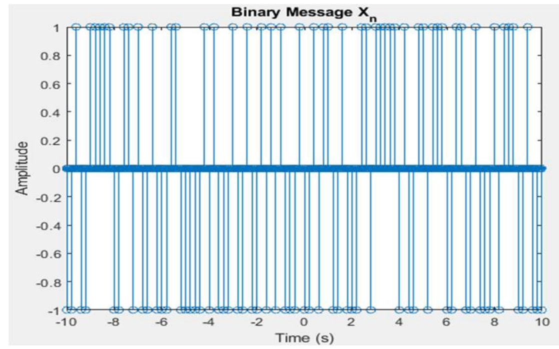

Figure 4: Binary Message

C. Modulating The Binary Message With Pulse Shapes

The binary message was then convolved with the pulse shapes to output the results in Figure 5.

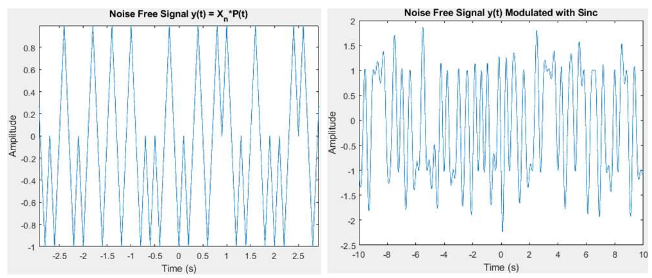

Figure 5: Binary signal modulated with triangular pulse (left) and sinc pulse (right)

D. Up-Converted to Higher Frequency

The message is now up converted to a higher frequency so that the signal can travel through the available mediums. In physics, the size of an antenna is inversely proportional to the frequency, so in practical communication systems, a high frequency band must be used because antenna infrastructure is built with this in mind. In this study the signals will be converted to 20 and 30 hertz respectively. This is done by multiplying the signal by a cosine of the chosen frequency.

E. Adding Noise

Some Gaussian noise is added to the signals to model real world conditions. This noise would accumulate as the signal travels through the medium. Figure 6 shows what the two signals look like after being up converted and infused with noise.

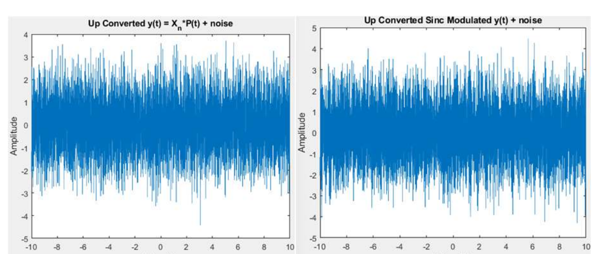

Figure 6: Signals Up-Converted and infused with Noise

F. Down-Conversion

In this stage, the signals have already passed through the medium and have arrived at their destination infused with noise. In this step, the signals are down converted from their higher frequencies by multiplying by the same cosine with 20 Hz for the triangular pulse signal and 30 Hz for the sinc modulated signal. The signals can be seen in Figure 7.

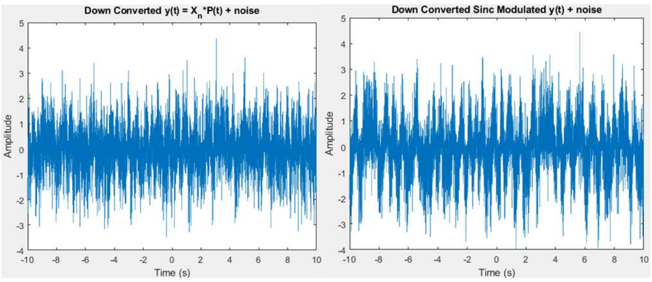

Figure 7: Down Converted Signals

G. PAM Simulation Using a Match Filter

By convolving the pulse shapes again with their respective down converted signals, the location of the bits is rediscovered. This method of detecting the presence of the original pulse shape in the signal through convolution is called a match filter and is considered the optimal linear filter for maximizing the signal to noise ratio (SNR).

The signal is then combed through using a for loop to find the bit values by looking at each bit interval and estimating whether it is closer to a 1 or a -1. The deciphered received signal after passing through the match filter can be seen in Figure 8 for the triangular pulse signal and in Figure 9 for the sinc pulse signal.

Both signals look very similar, but it can be noted that the sinc pulse modulated signal looks much smoother than the triangular pulse modulated signal.

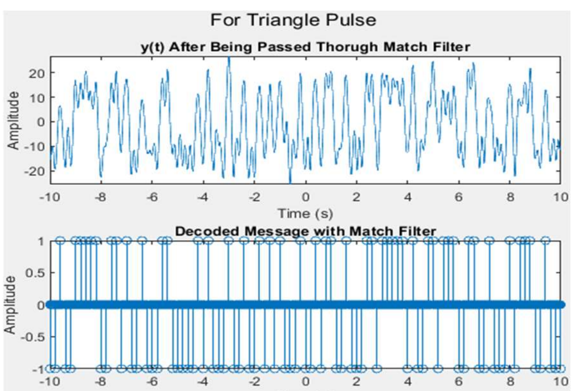

Figure 8: Triangular pulse signal processed with match filter

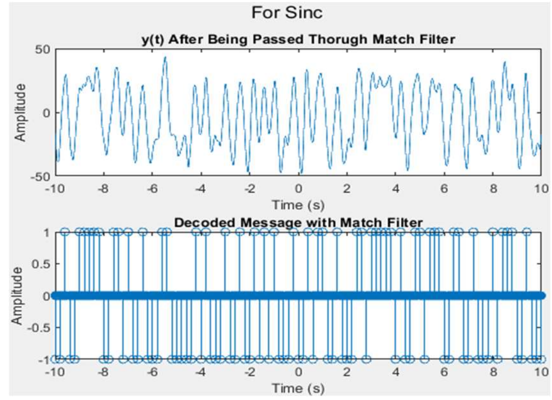

Figure 9: Sinc pulse signal processed with match filter

H. Comparing With Original Signal For Errors

Now that the received signal is back in binary, it is easier to compare with the original signal to see if any of the bits were flipped due to noise.

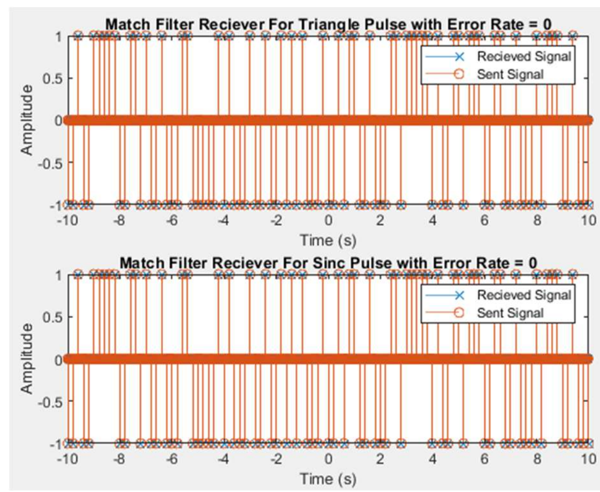

Figure 10: Comparison of the two received signals with original signals to see if any errors occurred

Figure 10 shows both the original signal and the received signals on one graph and it shows that the signals matched perfectly. With a bit error rate of 0, it can be said that this communication system can operate successfully with a high degree of confidence with the given parameters.

I. Sending an English Message

Sending an English message requires all the same previous steps with only two major steps added. First the English message must be converted into binary using a MATLAB function and then the binary must be processed so that the 0's become 1's and that that the 1's stay ones. This is easily done by filtering through with a for loop. The second major steps is reversing the first major step to convert the processed binary back into English.

When dealing with an English message, it is more imperative that all the sampling rates and periods line up because if even one interval is off, the system will not output the correct message and all the following bits would also be off. Not only is the accuracy of the bits important but also the spacing or the relative location/time of each bit must remain the same.

The message “We love ESE 351! :)” will be sent using triangular pulse modulation while the second message “We love ESE 351! :)” will be sent using sinc pulse modulation. Both messages are the same to remove doubt and ensure equality between the tests

With the all the parameters remaining the same as the previous tests, the messages are sent and received perfectly with no errors as to be expected given that the pulse shapes do not overlap, and the noise levels are relatively low. The bit rate is also currently 5 bits/sec which should be easy for the system to handle. Figure 11 shows a comparison between the sent and received messages.

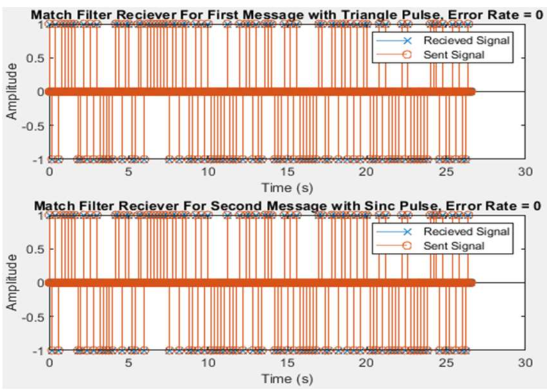

Figure 11: Shows Comparison of Both Messages

J. Increasing the Bit Rate

As the Bit rate increases, the error rate also increases. A gain of 4 on the bit rate making it 20 bits/second increases the error rate on both signals by about 20% but there are about 5% more errors in the triangular pulse modulated message as shown in Figure 12.

K. Changing Sampling Rate

When the sampling frequency f_s is increased a hundred-fold, the error rate on the sinc pulse modulated signal grows to be about 5% larger than the triangular pulse modulated signal as shown in Figure 13. Decreasing the sampling rate had a lot of effect when it was decreased below 250 samples/sec, which makes sense because that is exactly twice the highest frequency component which validates the Nyquist Theorem.

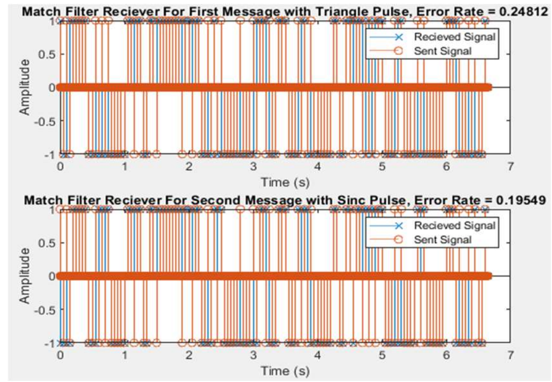

Figure 12: Showing the affect of Increasing Bit Rate 400%

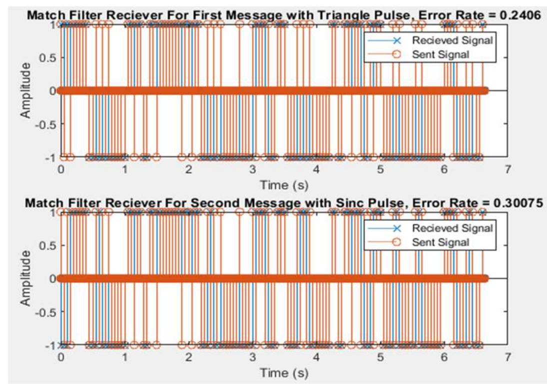

Figure 13: Effect of Increasing Bit Rate 400% and Sampling Rate 10,000%

L. Increasing The Noise

A gain of 10 on the Gaussian noise was tested and it was seen that on average the Triangular Pulse Modulated Message performed significantly worse than the Sinc Pulse Modulated signal. When the noise was increased dramatically to a gain of 50, there was little difference between the signals since this is a binary signal and the error rate will naturally peak around 50% which is what was observed in both signals.

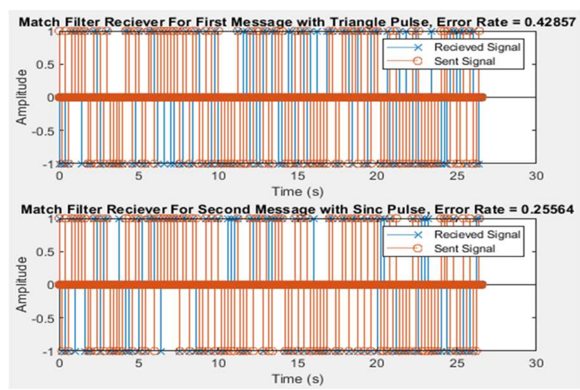

Figure 14: Effect of Gain on Gaussian Noise Being 10

IV. Conclusion

A Binary Pulse Amplitude Modulation System was successfully developed and implemented in MATLAB. The main parameters of the system were listed, and the Nyquist Sampling Theorem was correctly used to successfully send two messages across a simulated medium and the two messages were successfully reconstructed with no errors. How the different parameters of the system affect the performance of the system was successfully analyzed and conclusions were made. However successful the study was, it would benefit from a separate project where the system is built in a manner more inducive to efficient testing and thousands of trials. With a large number of trials, the effect of the parameters on the system and on each other can be better quantified and graphed. This study successfully discovered a qualitative relationship between the parameters, but future testing and experimentation will benefit the understanding gained and build relationships that can be used to build the ultimate efficient and reliable communication system.

V. Refrences

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