Lab 03: Simple Resistive Circuits

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I hereby certify that this report is my own original work.

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Abstract

In this paper the simplification of circuits into Thevenin and Norton equivalents is explored and shown to be a very powerful tool in circuit analysis. Empirical data collected from circuits built in this lab and the theoretical calculated values were compared to find very little difference between the two sets of data. This proves the validity of Thevenin and Norton Equivalent circuits as well as how accurate the calculated predictions are. Furthermore, the principle of superposition was empirically explored and validated by building a circuit and comparing how each independent source affects an element in the circuit. It was validated that each circuit variable of an element can be measured as the sum of the values inflicted from each independent power source. These simplifications showed how accurate these theorems are and how practical and efficient they are in the lab setting as well as in theoretical calculations.

Introduction

Circuits can be very complicated especially when they are layered or connected to each other. If there are 5 big complex circuits, and the goal is to connect them and calculate the total resistance for example, it would be a very time-consuming process as well as include many steps which increases the chance error in the calculations. Therefore, Thevenin's Theorem states that complex circuits can be simplified to either a voltage source with the total resistance or a current source in parallel with the total resistance in a circuit. In this manner, no matter how big or complex a circuit is, it can be simplified into a much smaller simpler circuit with the important circuit variables voltage, current, and resistance defined at specific probes/nodes [3]. Equation one showcases the relationship between the three values and is basically an extension of Ohm's Law.

$$V_{Th} = i_N \times R_{Th} \tag{1}$$

From the equation it can also be concluded that only two of the unknowns must be found in order to find the third and final circuit variable. The node-voltage method was also validated using empirical data from a circuit and Kirchhoff's junction rules were found to be very accurate and precise. The Superposition Principle will also be explored and the power of isolating power sources one at a time to calculate the equivalent value at a specific circuit element is a very powerful proposition and shows up a multitude of times in physics [2]. An example of the superposition principle is showcased in Figure 0 which was brought from the textbook [2].

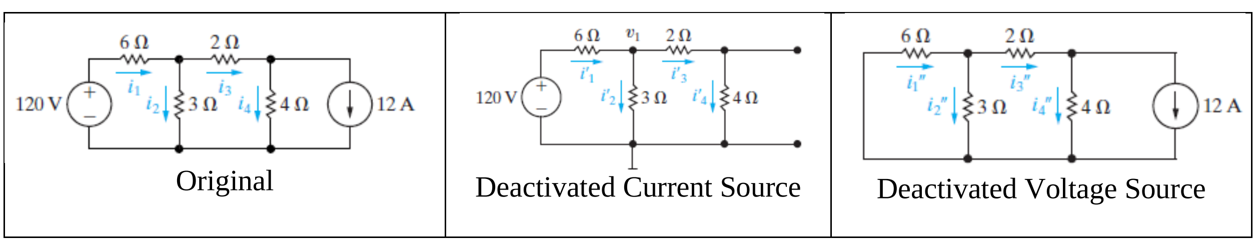


Figure 0: Shows how the Superposition principle can be used to isolate sources

Apparatus and Procedures

This lab consists of a series of experiments. Each experiments equipment, setup, and purpose are listed in the order that they were performed in the lab [3].

Experiment 1

Equipment Used:

- DMM
- Oscilloscope
- Keithley DC power supply
- Breadboard
- 6 banana cables
- 6 alligator clips
- Two resistors: 220 Ω
- Two resistors: 51 Ω
- Four resistors: 100 Ω

Setup

1. Measure the resistance of each resistor to find their exact values.
2. Connect the circuit in figure 1 on the breadboard using the resistors measured and the DC power supply to supply the 9 V.

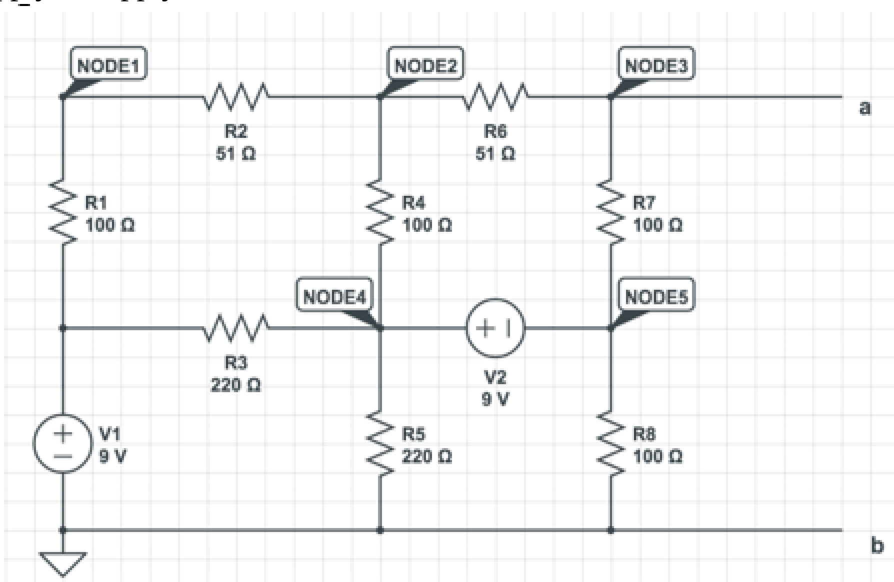


Figure 1: Circuit that will be analyzed using the node voltage method

3. Measure the node voltages at each node.
4. Measure the node voltages caused by each voltage source individually by turning off 1 voltage source while keeping the other on.

Experiment 2

Equipment Used:

- DMM
- Breadboard
- 3.5V DC voltage source
- Three resistors: 10Ω, 100Ω, 220Ω
- Jumper wires
- Mini-grabber cables

Setup

1. Measure the resistance of each resistor so the exact value is known.
2. Create the circuit shown in Figure 2 using the resistors measured and the DC power supply to supply the 3.5 V.D

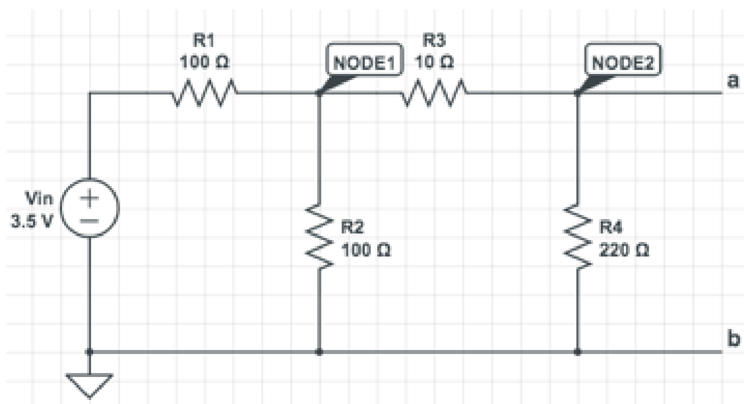


Figure 2: The second circuit that will be built for finding the Thevenin voltage and Resistance

3. Use the DMM to find the voltage at the terminals ab.
4. Deactivate the voltage source and calculate the Thevenin resistance at the terminals.

Analysis and Results

Experiment 1: Determining and verifying node-voltages.

In this lab, a lot of empirical data was collected to support the findings proposed in this paper. Below are the various tables and pictures that were collected during the lab and were analyzed to support the conclusion of this paper.

Table C1: Measured values of the resistors

Element	Measured value	Element	Measured value
R1	98.29	R5	217.74
R2	50.71	R6	50.60
R3	218.40	R7	99.00
R4	99.00	R8	98.65

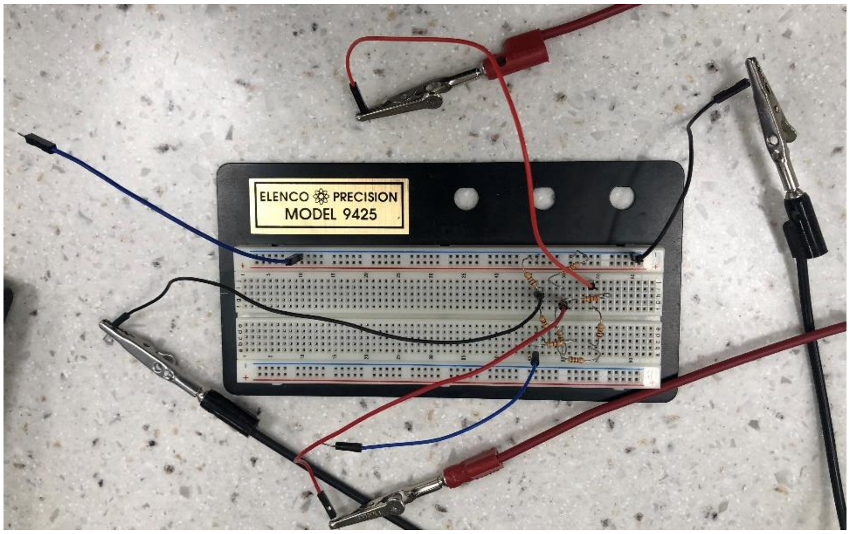
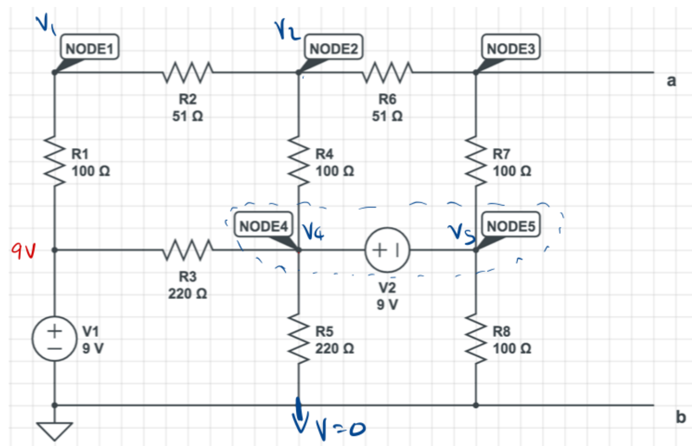


Figure 3: The circuit from figure 1 built on the bread board



Supernode:

$$\frac{V_4-9}{220} + \frac{V_4-V_2}{100} + \frac{V_4}{220} + \frac{V_5-V_3}{100} + \frac{V_5-0}{100} = 0$$

Voltage drop across supernode:

$$V_4 - V_5 = 9$$

for V_2 $$\frac{V_2-9}{151} + \frac{V_2-V_4}{100} + \frac{V_2-V_3}{51} = 0$$

for V_3 $$\frac{V_3-V_2}{51} + \frac{V_3-V_5}{100} = 0$$

for V_4 & V_5 $$\frac{V_4-V_2}{100} + \frac{V_4-9}{220} + \frac{V_4}{220} + \frac{V_5}{100} + \frac{V_5-V_3}{100} = 0$$

$$\frac{V_1-9}{100} + \frac{V_1-V_2}{51} = 0$$

$$\frac{V_2-V_1}{51} + \frac{V_2-V_3}{51} + \frac{V_2-V_4}{100} = 0$$

$$\frac{V_3-V_2}{51} + \frac{V_3-V_5}{100} = 0$$

$V_1 = 6.83$ V
$V_2 = 5.72$ V
$V_3 = 3.45$ V
$V_4 = 8.00$ V
$V_5 = -1.00$ V

Figure 4: Shows the calculations for the node voltages using the node voltage method

Table C2: Measured voltage across the 5 nodes

Node number	Measured voltage (V)	Calculated value (V)	% Error
1	6.81	6.83	0.29 %
2	5.70	5.72	0.35 %
3	3.42	3.45	0.87 %
4	7.97	8.00	0.38 %
5	-1.00	-1.00	0 %

It can be seen in Figure C2 that the measured voltages and the calculated voltages are very close to one another with less than 1 % error between the measured and the calculated values. This shows that node voltage method is effective node voltages with out needing to take empirical measurements in most cases, unless extreme accuracy/ precision is expected.

Experiment 2: Testing the superposition Theorem.

In this part of the lab, the superposition principle will be empirically tested by measuring the node voltages with the voltage source V2 disabled and then with V1 disabled. Adding the two measurements should yield the same numbers found in Figure 4 and Table C2.

The voltage sources were deactivated by placing a wire instead of the source, in Figueres 5 and 7 it is the orange wire.

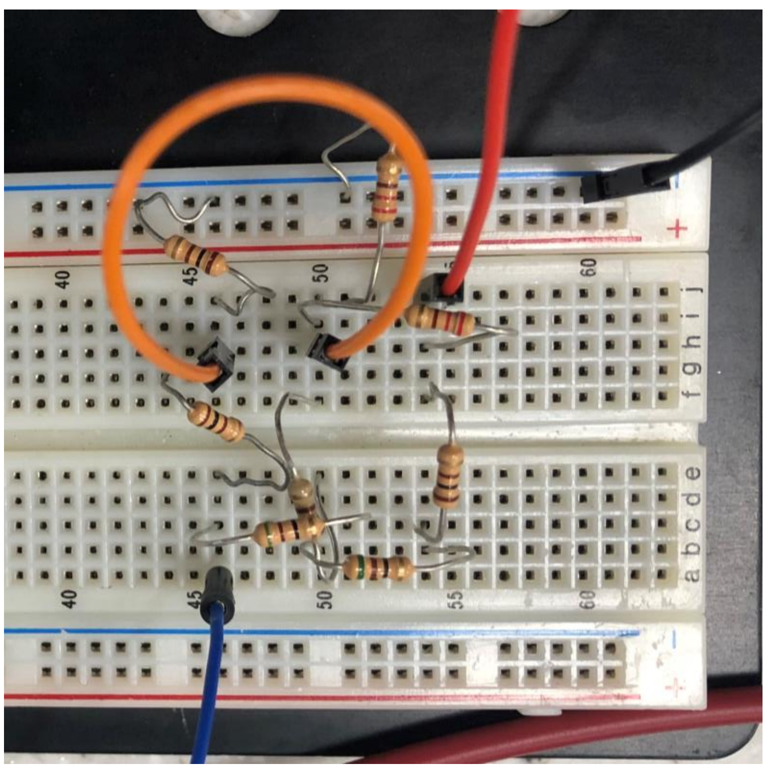


Figure 5 Circuit with V2 disabled

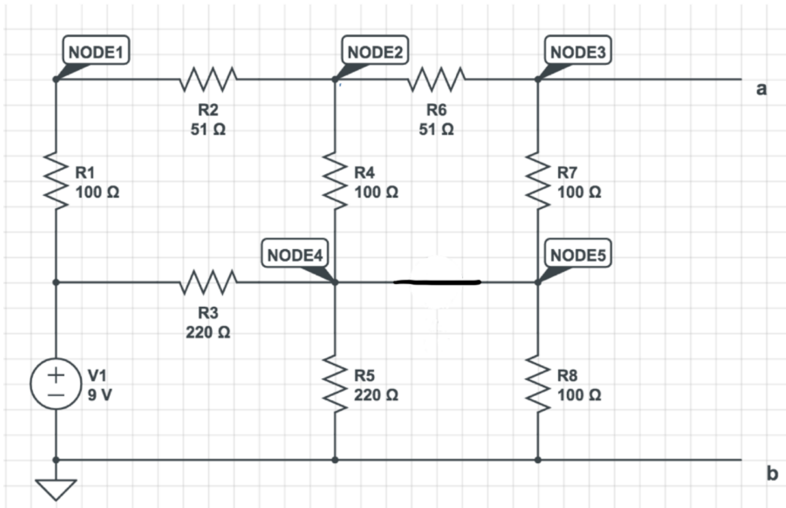


Figure 6: shows a circuit diagram with voltage source V2 disabled

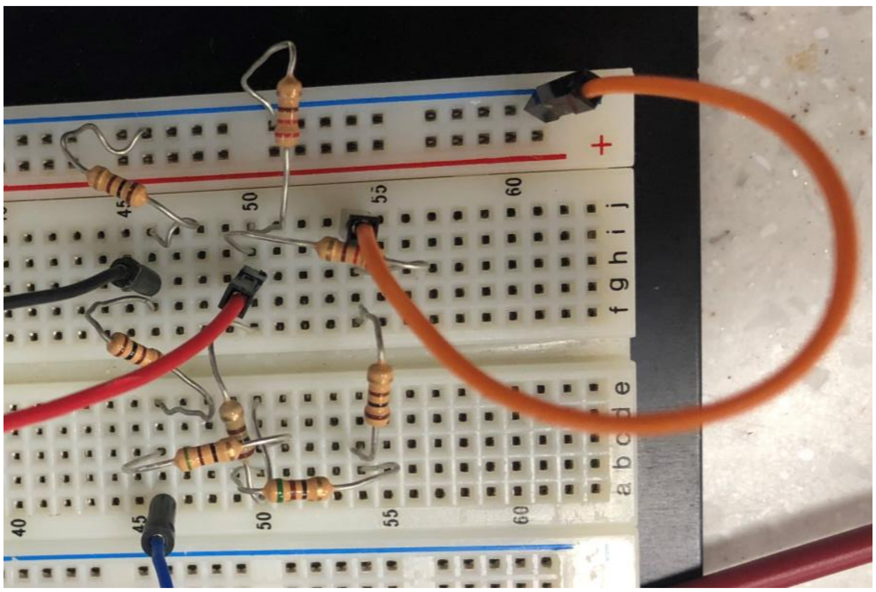


Figure 7: Shows the circuit with voltage source V1 disabled

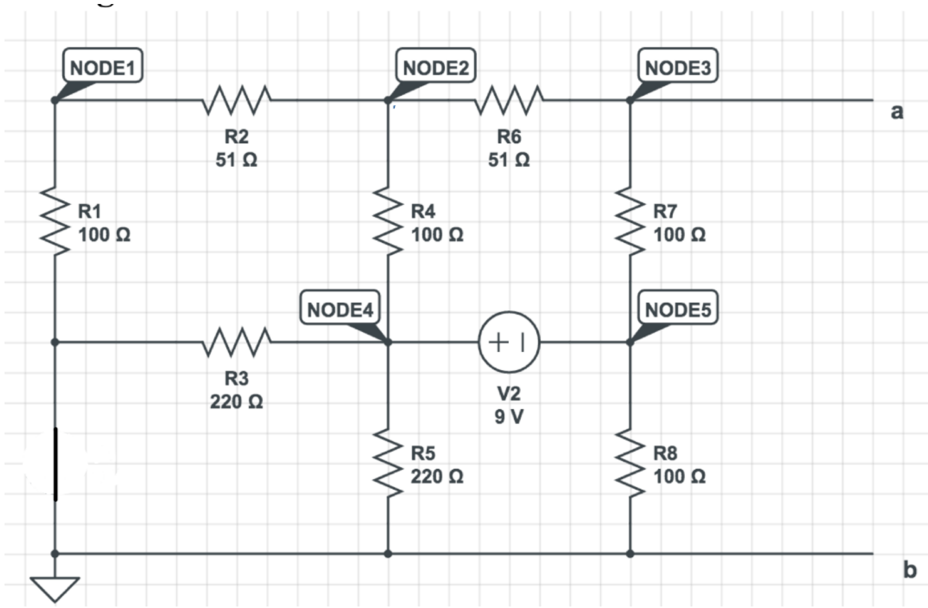


Figure 8: Shows circuit diagram with voltage source V1 disabled

Table D1: Voltages Measured due to only 1 Voltage Source at a Time

Node number	Voltage with V1 only (V)	Voltage with V2 only (V)	Sum of voltages from V1 & V2	Node voltages from section C	% diff.
1	6.38 V	0.44 V	6.82	6.81	0.15 %
2	5.05 V	0.67 V	5.72	5.70	0.35 %
3	4.52 V	-1.08 V	3.44	3.42	0.58 %
4	3.50 V	4.48 V	7.98	7.97	0.13 %
5	3.50 V	-4.49 V	-0.99	-1.00	1 %

As can be seen from Table D1, the percent error between the sum of the two independent voltages and the node voltage found in the previous part of the lab were found to be on average less than 1 %. The difference of the values is most likely attributed to discrepancies in the DMM, but overall, these results verify that the superposition principle does indeed work and is very precise.

Experiment 3: Thevenin Equivalent Circuits.

In part 3 of this lab, a new circuit was built in order to test the concept of the Thevenin Equivalent [2]. The circuit built is show in Figure 9.

Table E1: Measured Values of the 3 resistors in ohms

Element	Measured value	Element	Measured value
R1	99.52	R3	10.03
R2	98.50	R4	217.47

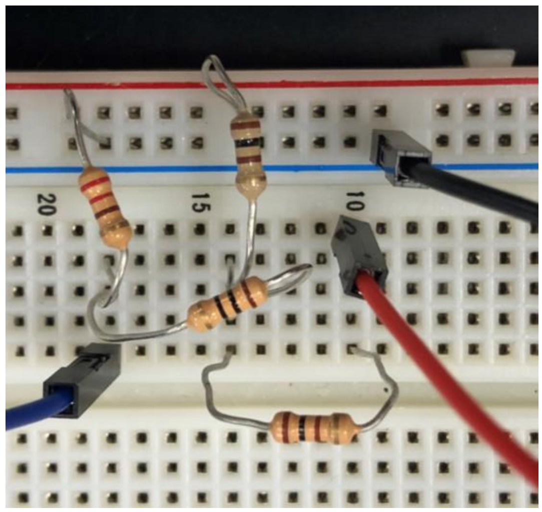


Figure 9: Shows the new constructed circuit

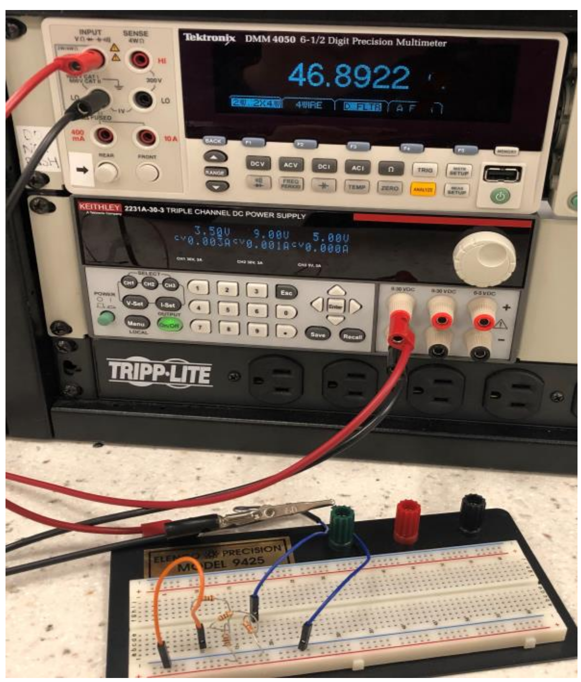


Figure 10: Shows the circuit as well the Thevenin Resistance reading on the DMM

Table E2: The Measured and Calculated Values of the Thevenin Voltage and Resistance

	Measured value(V)	Calculated value(V)	% Error
V_{Th}	1.36 V	1.38 V	1.45 %
R_{Th}	46.89	47.14	0.53 %

From Table E2, it can be seen that the measured and calculated values for the Thevenin Voltage and Resistance were very close and the percent error was less than 2 %. This verifies the theorem and showcases its power of simplification and efficiency. In Figure 11, it's shown how the circuit in Figure 10 was transformed into a much simpler circuit.

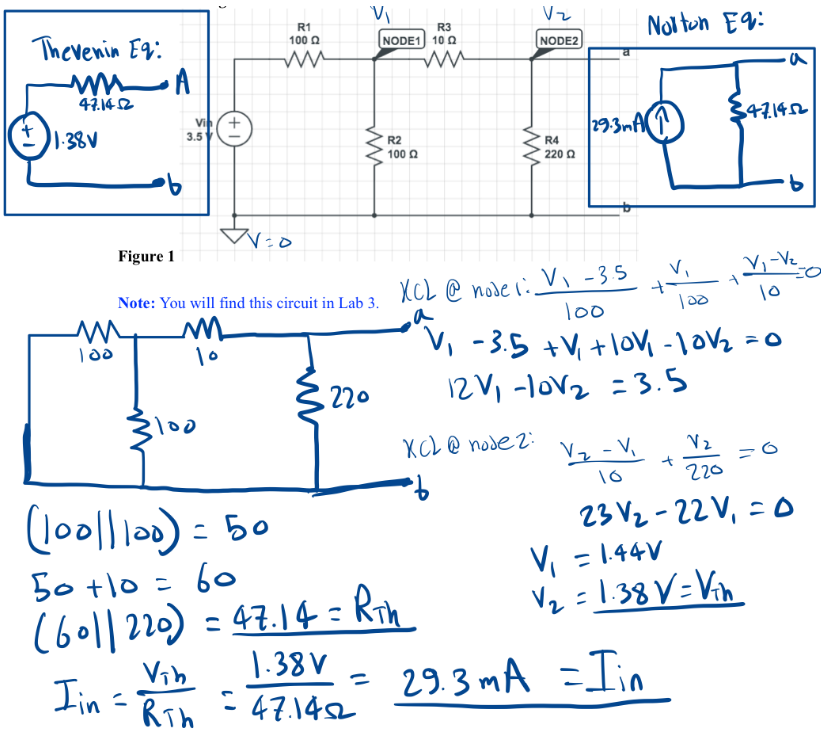


Figure 11: Shows the Thevenin and Norton Equivalent Circuits as well as their derivations

Conclusion

It was empirically proven that the Node Voltage Method, the Superposition Principle and the Thevenin Theorem all hold true under real world conditions. The use of these methods increases the efficiency of circuit analysis and gives very good approximation of what the values would be using a DMM. In most of the experiments, the percent error between the calculated and measured values was well under 1 %. To improve this lap, Instead of only measuring the Thevenin voltage, it would also be helpful to gain experience measuring the current at the nodes as well as using current sources in addition to voltage sources. This would add another level of complexity to the lab and aid in the understanding of how the different sources compare specially when using the Superposition Principle [2].

References

[1] *Percentage difference calculator*. CalculatorSoup. (n.d.). Retrieved February 15, 2022, from https://www.calculatorsoup.com/calculators/algebra/percent-difference-calculator.php

[2] Nilsson, J. W., & Riedel, S. A. (2019). *Electric Circuits* (10th ed.). Pearson Education, Inc.

[3] Nussinov, T., Lab 1 Handout, Spring 2021, "Simple Resistive Circuits"

Note: I worked alone in the lab and on the report due to my partner missing for personal reasons