

Drone Moment of Inertia Using Bifilar Torsion Pendulum

Introduction:

The primary objective of this lab was to set up a bifilar torsion pendulum to empirically determine the crazy fly drone's moment of inertia along its three axis, (I_x, I_y, I_z) . An object's moment of inertia is essentially an object's resistance to changes in its rotational motion, which plays an important role in the stability and performance along the different axis. With the goal of designing a simple flight controller for the crazy fly drone, it's integral to calculate the moments of inertia about each axis and understand how these values affect the stability and maneuverability of the drone.

Theory:

The Bifilar Torsion Pendulum is a mechanism/procedure that allows for the measurement of an object's moment of inertia by observing its oscillatory motion when suspended by its respective axis by two strings. The period of oscillations to be observed is influenced by multiple factors present in equation (1) below.

$$T = \sqrt{\frac{16\pi^2 l I}{m d^2 g}} \quad (1)$$

where,

- T is the oscillation period
- l is the length of the strings (from hanger to object).
- d is the distance between the strings.
- g is the gravitation acceleration.
- I is the rotational inertia along the respective rotational axis.
- m is the mass of the object.

Since T is what is being observed, the equation can be solved for the moment of inertia I as can be seen in equation 2.

$$I = \frac{m d^2 T^2 g}{16\pi^2 l} \quad (2)$$

This equation will have to be solved for each of the three rotational axis but some of the parameters will stay the same such as m and g .

Procedure:

The steps taken for the calculation of the moment of inertia (I_x, I_y, I_z) will now be listed in order of completion [1].

1. Weigh your drone and record the mass
2. Attach two strings to the drone along the positive x axis, so that the strings are parallel to the x-axis
3. Make sure the strings are vertical compare to the table
4. Measure the distance between the two strings (d) and the length of the pendulum (l).
5. Rotate the drone away from its equilibrium position and hold. Be mindful of your endpoint (keep your rotation less than 20 degrees)
6. When partner is ready, release the drone and start the timer
7. Stop the timer after 10 oscillations; calculate and record average oscillation period.
8. Repeat the above steps for an extra run
9. Switch axis, repeat all above steps for axis y and z

Results:

The results of following the procedure and after completeing the calculations is outlined below:

$$m = 0.0319 \text{ kg} , g = 9.81 \frac{m}{s^2}$$

- $I_x = 1.3 \times 10^{-5}$
- $I_y = 1.6 \times 10^{-5}$
- $I_z = 3.4 \times 10^{-5}$

For the x-axis:	For the y-axis:	For the z-axis:
Distance between strings: 5.5 cm	Distance between strings: 5.5 cm	Distance between strings: 7.5 cm
Length of strings: 35.8 cm	Length of strings: 28.8 cm	Length of strings: 28 cm
Time run 1: 9.32 s	Time run 1: 8.93 s	Time run 1: 9.28 s
Time run 2: 8.26 s	Time run 2: 8.68 s	Time run 2: 9.08 s
Time avg: 8.79 s	Time avg: 8.805 s	Time avg: 9.18 s

Conclusion:

This workshop successfully demonstrated the application of the Bifilar Torsion Pendulum method in determining the moments of inertia for the crazy fly drone. While these values are often provided in the data sheet for most commercial drones, there are variations in parameters that do occur that will negatively impact the preformance and robustness of the drone if it is not taken into account. By completing this experiment, the data sheet moment of inertia values can be verified and refined if need be. Now these values can be used in the development of flight software for the crazy fly drone.

References:

[1] Wu, W., Warner, B., & Rijsinghani, S. (2022). Bifilar Pendulum Experiment for Determining Moments of Inertia in Crazyflie Drones. Workshop 1, ESE 4481, October 18.