## ESE 4481 Assignment #6 / Seif Elkhashab / 04-01-2024

## Least Squares

**The speed of the CrazyFly 2.1 four propellers is $n_1$, $n_2$, $n_3$ , and $n_4$. There is 92 mm of distance diagonal distance between opposite motors (e.g., between motor 1 and 3). The propellers are 45 mm in diameter. A table of data of the net thrust from 4 propellers is provided with the following data:**

| Amps | Thrust (g) | Voltage | PWM (%) | RPM |
|---|---|---|---|---|
| 0.24 | 0.0 | 4.01 | 0 | 0 |
| 0.37 | 1.6 | 3.98 | 6.25 | 4485 |
| 0.56 | 4.8 | 3.95 | 12.5 | 7570 |
| 0.75 | 7.9 | 3.92 | 18.75 | 9374 |
| 0.94 | 10.9 | 3.88 | 25 | 10885 |
| 1.15 | 13.9 | 3.84 | 31.25 | 12277 |
| 1.37 | 17.3 | 3.80 | 37.5 | 13522 |
| 1.59 | 21.0 | 3.76 | 43.25 | 14691 |
| 1.83 | 24.4 | 3.71 | 50 | 15924 |
| 2.11 | 28.6 | 3.67 | 56.25 | 17174 |
| 2.39 | 32.8 | 3.65 | 62.5 | 18179 |
| 2.71 | 37.3 | 3.62 | 68.75 | 19397 |
| 3.06 | 41.7 | 3.56 | 75 | 20539 |
| 3.46 | 46.0 | 3.48 | 81.25 | 21692 |
| 3.88 | 51.9 | 3.40 | 87.5 | 22598 |

The Function from PWM to RPM:
RPM = f(PWM) = 3.472 pwm + 85.365

**(1) Estimate the propeller's thrust coefficient. Assume sea level air density.**

The Thrust Coeffecient:
$C_T = 0.1732223315$

**(2) Estimate the propeller's power coefficient. Assume sea level air density.**

The Power Coeffecient:
$C_P = 0.3119637027$

**(3) Estimate the propeller's torque coefficient. Assume sea level air density.**

The Torque Coeffecient:
$C_{Q} = 0.0535322322$

**(4) Determine pitching moment produced by each motor as a function of PWM %**

The Pitching Moment as a function of PWM:
Pitching Moment = $0.000000040026522032090009621612748185075\ (3.472\ \text{pwm} + 85.365)^2$

**(5) Determine the rolling moment produced by each motor as a function of PWM %**

The Rolling Moment as a function of PWM:
Rolling Moment = $0.000000040026522032090009621612748185075\ (3.472\ \text{pwm} + 85.365)^2$

**(6) Determine yawing moment produced by each motor as a function of PWM %**

The Yawing Moment as a function of PWM:
Yawing Moment = $0.0000000121007979827900138794913224344487\ (3.472\ \text{pwm} + 85.365)^2$

**(7) Estimate the PWM % of the four propellers (assuming they all maintain the same constant speed) to produce a thrust equal and opposite to gravity (trim to 1g).**

PWM % required for trim:
PWM % = 54.8666

**An engineer seeks to estimate the torque coefficient $c_t$ of a propeller with a known moment of inertia, 1 $kg \cdot m^2$. The angular velocity of the propeller is measured $\omega(t)$ and so is the applied torque $T_{app}$. The propeller has a coefficient of torque that is unknown. Assume that the propeller has diameter of 1 $m$ and that the air has density 1.225 $kg/m^3$. Download the provided data set on Canvas and estimate the propeller's coefficient of torque using a first-order transfer function fit as follows:**

$$\dot{\omega}(t) = T_{app} - k\omega(t)$$

**Note that since the coefficient of torque is defined from $\rho C_p D^5 \left(\frac{\omega}{2\pi}\right)^2$ then $k$ must satisfy**

$$k = 2\rho C_t D^5 \left(\frac{1}{2\pi}\right)^5$$

The Estimated Torque Coeffecient:
$C_{t} = 788.123$